## AP Statistics - Chapter 10 Practice Free Response Test: Comparing Two Populations

1. A study of iron deficiency among infants compared samples of infants following different feeding regimens. One group contained breast-fed infants, while the children in another group were fed a standard baby formula without any iron supplements. Here are the results on blood hemoglobin levels at 12 months of age.

| Group | $\boldsymbol{n}$ | $\boldsymbol{x}$ | $\boldsymbol{s}$ |
| :--- | :--- | :--- | :--- |
| Breast-fed | 23 | 13.3 | 1.7 |
| Formula | 19 | 12.4 | 1.8 |

(a) Is there significant evidence that the mean hemoglobin level is higher among breast-fed babies? Give appropriate statistical evidence to support your conclusion.

Do all of the following:
a. Identify the parameters of interest. Then state the appropriate hypotheses.
b. Verify conditions for carrying out a significance test.
c. Calculate the test statistic and the $P$-value.
d. What conclusion would you draw?
2. An association of Christmas tree growers in Indiana sponsored a sample survey of 500 randomly selected Indiana households to help improve the marketing of Christmas trees. One question the researchers asked was "Did you have a Christmas tree this year?"

Respondents who had a tree during the holiday season were asked whether the tree was natural or artificial. Respondents were also asked if they lived in an urban area or in a rural area. Of the 421 households displaying a Christmas tree, 160 lived in rural areas and 261 were urban residents. Here are the data:

| Population | $n$ | $X$ (natural) |
| :--- | :--- | :--- |
| 1 (rural) | 160 | 64 |
| 2 (urban) | 261 | 89 |

Do these data provide evidence of a significant difference in the proportion of rural and urban Indiana residents who had a natural Christmas tree this year? Perform an appropriate test to answer this question.
a. Identify the parameters of interest. Then state the appropriate hypotheses.
b. Verify conditions for carrying out a significance test.
c. Calculate the test statistic and the $P$-value.
d. What conclusion would you draw?
3. In the 2001 regular baseball season, the World Series Champion Arizona Diamondbacks played 81 games at home and 81 games away. They won 48 of their home games and 44 of the games played away. We can consider these games as samples from potentially large populations of games played at home and away.
a. Identify the populations and parameters of interest.
b. Construct and interpret a $90 \%$ confidence interval for the difference between the proportion of games that the Diamondbacks win at home and the proportion that they win when on the road.
c. Most people think that it is easier to win at home than away. Use the confidence interval from part $b$ to determine whether this is true for the Arizona Diamondbacks.

## AP Statistics - Chapter 10 Practice Free Response Test - SOLUTIONS

## Problem \#1

1. (b) 2. (a) 3. (d) 4. (c) 5. (b) 6. (b) 7. (a) Step 1: Let breast-fed infants be population 1, and formula-fed infants be population 2. $\mu$ is the mean hemoglobin level. Our hypotheses are $H_{0}: \mu_{1}=$ $\mu_{2}\left(\right.$ or $\mu_{1}-\mu_{2}=0$ ) and $H_{3}: \mu_{1}>\mu_{2}$ (or $\mu_{1}-\mu_{2}>0$ ). Step 2: SRS-We are not told that either sample is an SRS (or even random). This may limit our ability to generalize. Normality-The combined sample size is more than 30 , so the $t$ procedures for two samples should be robust against potential skewness, provided that there are no extreme outliers. Independence-We must assume that the two samples of measurements are independent. Step 3: The test statistic is: $t=\frac{13.3-12.4}{\sqrt{\frac{1.7^{2}}{23}+\frac{1.8^{2}}{19}}}=1.6537$.
With $\mathrm{df}=18,0.05<P$-value $<0.10$. By TI calculator, $\mathrm{df}=37.5876$, and the $P$-value $=0.0533$.
Step 4: There is only marginal evidence to reject $H_{0}$. We are reluctant to conclude that the mean hemoglobin level is higher for breast-fed babies. These results are so close to the commonly accepted standard of $\alpha=0.05$ that we would want to replicate this experiment before announcing any significant results. (b) $\mathrm{A} 95 \% \mathrm{CI}$ for $\mu_{1}-\mu_{2}$ is

## Problem \#2

We want to test $H_{0}: p_{1}=p_{2}$ vs. $H_{a}: p_{1} \neq p_{2}$.
decker buses. 9. (a) Step 1: Let $p_{1}=$ the proportion of all rural Indiana residents who had a natural Christmas tree this year and $p_{2}=$ the proportion of all urban Indiana residents who had a natural Christmas tree this year. Step 2: SRS—We are told that the sample involved "randomly selected Indiana households," so we should be safe generalizing our results to the populations of interest. Independence-The population of rural Indiana residents is at least $10(160)=1600$ and of urban Indiana residents at least $10(261)=2610$. Normality-We are safe using the two proportion $z$ procedures since the counts of successes (natural tree) and failures (not a natural tree) are at least 10 in both samples. Step 3: The $95 \% \mathrm{CI}$ for $p_{1}-p_{2}$ is
P1-hat $=64 / 160=0.4$, P2-hat $=89 / 261=0.341$, Pc-hat $=(64+89) /(160+261)=0.363$
$\mathrm{Z}=\frac{.4-.341}{\sqrt{\frac{4(.6)}{160}+\frac{341(.659)}{261}}}=\mathbf{1 . 2 2 2}$ and p -value $=.222$.
Conclusion: Since the p-value is greater than .05 , we fail to reject Ho, which means we cannot conclude that there is a significance difference in the proportion of rural and urban Indiana residents who had a natural Christmas tree this year

## Problem \#3

1. Let $p_{1}=$ the proportion of all home games won by the Arizona Diamondbacks and $p_{2}=$ the proportion of away games won. The standard error for a confidence interval estimate of $p_{1}-p_{2}$ is $\sqrt{\frac{(0.593)(0.407)}{81}+\frac{(0.543)(0.457)}{81}}=0.0777$. 2. We are willing to treat these samples of 81 games as independent SRSs from the respective populations of interest. The counts of successes and failures are all at least 5 , so the $z$ procedures should be fairly accurate. Our $90 \% \mathrm{CI}$ is
$\left(p_{1}-p_{2}\right) \pm z^{*} \mathrm{SE}=(0.593-0.543) \pm 1.645(0.0777)=(-0.079,0.177)$. We are $90 \%$ confident that the difference in the proportion of Diamondbacks' home and away wins is between winning $7.9 \%$ more away games and winning $17.7 \%$ more home games. 3. Step 1: Our hypotheses are $H_{0}: p_{1}=p_{2}$ vs.
c) Since the interval contains 0 , home and away winning percentage could likely be the same for the Arizona Diamondbacks. A home-field advantage is not evident from this data.
